EASTBURY FARM WRITTEN CALCULATIONS POLICY

Introduction

This calculation policy has been written to ensure a consistent approach to the teaching of written calculation.

Our aim is to develop children who are confident Mathematicians, who have the ability to reason mathematically and can apply their Maths skills in a wide range of contexts.
The methods taught will be in line with the new Programmes of Study and will help develop children's fluency, reasoning and problem solving.

'To ensure that all pupils become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.'

National Curriculum 2014

Through the policy, we aim to link key manipulatives and representations so that the children can be vertically accelerated through each strand of calculation. As children move at the pace appropriate to them, teachers will be presenting strategies and equipment appropriate to children's level of understanding.

We will also be placing emphasis on using visualisation to help understand and develop each of the four operations. In producing such visualisation, the child is able to identify and internalise the key components and the relationship between them.

The importance of mental mathematics

While this policy focuses on written calculations in mathematics, we recognise the importance of the mental strategies and known facts that form the basis of all calculations. The following checklists outline the key skills and number facts that children are expected to develop throughout the school.

To add and subtract successfully, children should be able to:
· Recall all addition pairs to 9 + 9 and number bonds to 10
· Recognise addition and subtraction as inverse operations
· Add mentally a series of one digit numbers (e.g. 5 + 8 + 4)
· Add and subtract multiples of 10 or 100 using the related addition fact and their knowledge of place value (e.g. 600 + 700, 160 — 70)
· Partition 2 and 3 digit numbers into multiples of 100, 10 and 1 in different ways (e.g. partition 74 into 70 + 4 or 60 + 14)
· Use estimation by rounding to check answers are reasonable

To multiply and divide successfully, children should be able to:
· Add and subtract accurately and efficiently
· Recall multiplication facts to 12 x 12 = 144 and division facts to 144 ÷ 12 = 12
· Use multiplication and division facts to estimate how many times one number divides into another etc.
· Know the outcome of multiplying by 0 and by 1 and of dividing by 1.
· Understand the effect of multiplying and dividing whole numbers by 10, 100 and later 1000
· Recognise factor pairs of numbers (e.g. that 15 = 3 x 5, or that 40 = 10 x 4) and increasingly able to recognise common factors
· Derive other results from multiplication and division facts and multiplication and division by 10 or 100 (and later 1000)
· Notice and recall with increasing fluency inverse facts
· Partition numbers into 100s, 10s and 1s or multiple groupings
· Understand how the principles of commutative, associative and distributive laws apply or do not apply to multiplication and division
· Understand the effects of scaling by whole numbers and decimal numbers or fractions
· Understand correspondence where n objects are related to m objects
· Investigate and learn rules for divisibility

Progression in Addition and Subtraction

Addition and subtraction are connected.

Part	Part
Whole	

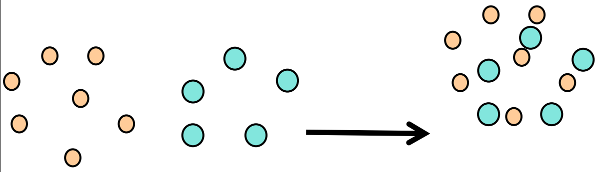
Addition names the whole in terms of the parts and **subtraction** names a missing part of the whole.

ADDITION

Combining two sets (aggregation)

Putting together – two or more amounts or numbers are put together to make a total

7 + 5 = 12



Count one set, then the other set. Combine the sets and count again. Starting at 1.

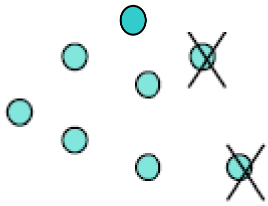
Counting along the bead bar, count out the 2 sets, then draw them together, count again. Starting at 1.

SUBTRACTION

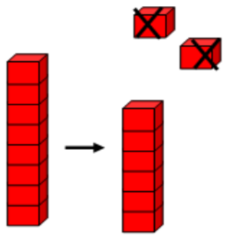
Taking away (separation model)

Where one quantity is taken away from another to calculate what is left.

8 – 2 = 6



Multilink towers- to physically take away objects.



END OF RECEPTION:

- **EYFS: 40-60+ MONTHS: Finds the total of items in two groups by counting all of them.**
  In practical activities and discussion, begin to use the vocabulary involved in adding and subtracting.
- **ELG11: Using Quantities, Numicon and objects they add and subtract two single digit numbers and count on and back to find the answer.**

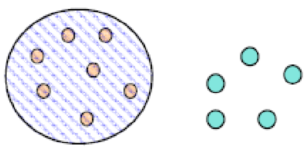
ADDITION

Combining two sets (augmentation)

This stage is essential in starting children to calculate rather than counting

Where one quantity is increased by some amount. Count on from the total of the first set, e.g. put 3 in your head and count on 2. Always start with the largest number.

Counters:



Start with 7, then count on 8, 9, 10, 11, 12

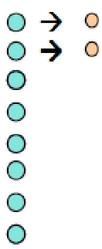
SUBTRACTION

Finding the difference (comparison model)

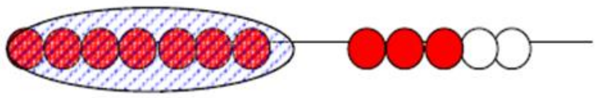
Two quantities are compared to find the difference.

8 – 2 = 6

Counters:



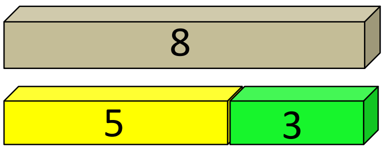
Bead strings:



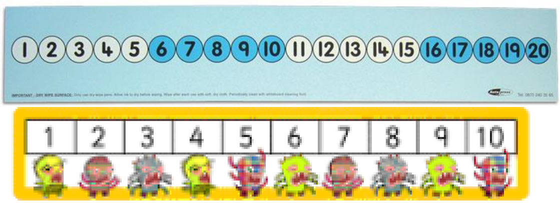
Make a set of 7 and a set of 5. Then count on from 7.

Multi-link Towers:

Cuisenaire Rods:

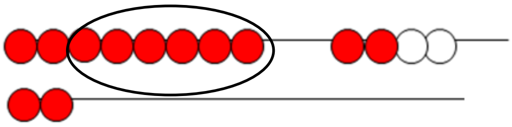


Number tracks:

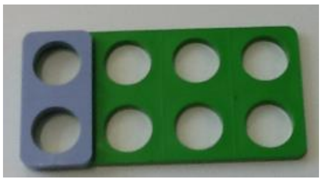


Start on 5 then count on 3 more

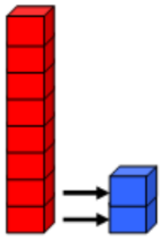
Bead strings:



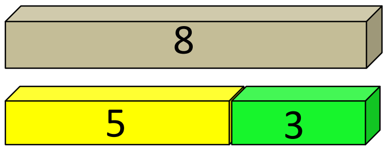
Make a set of 8 and a set of 2. Then count the gap.



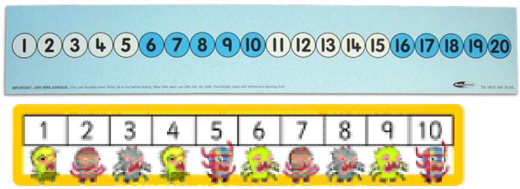
Multi-link Towers:



Cuisenaire Rods:



Number tracks:



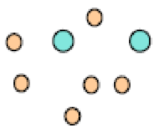
Start with the smaller number and count the gap to the larger number.

1 set within another (part-whole model)

The quantity in the whole set and one part are known, and may be used to find out how many are in the unknown part.

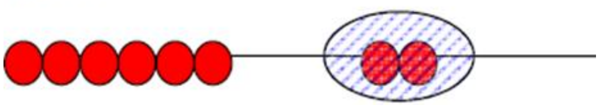
8 – 2 = 6

Counters:



Bead strings:

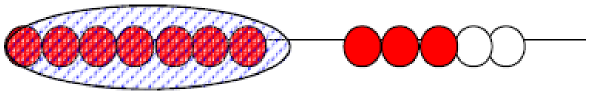
8 – 2 = 6



Bridging through 10s

This stage encourages children to become more efficient and begin employ known facts.

Bead string:



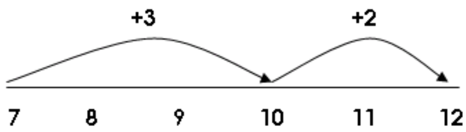
7 + 5 is decomposed/partitioned into 7 + 3 + 2.

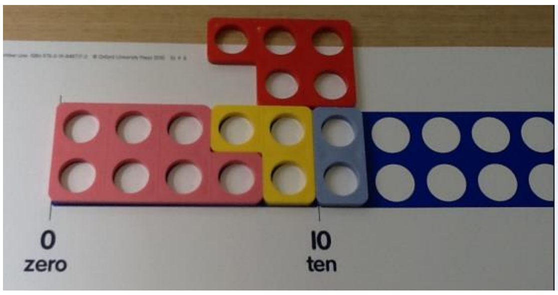
The bead string illustrates 'how many more to the next multiple of 10?' (the children should identify how their number bonds are being applied) and then 'if we have used 3 of the 5 to get to 10, how many more do we need to add on? (ability to decompose/partition all numbers applied)

Number track:

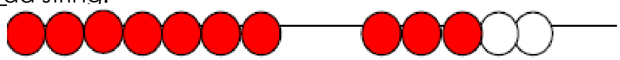
Steps can be recorded on a number track alongside the bead string, prior to transition to number line.

Number line





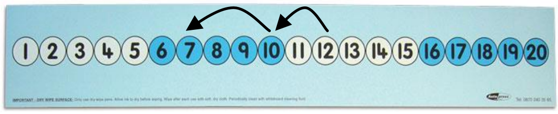
Bead string:



12 – 7 is decomposed/partitioned in 12 – 2 – 5.

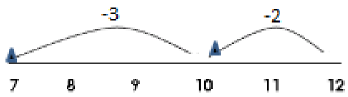
The bead string illustrates 'from 12 how many to the last/previous multiple of 10?' and then 'if we have used 2 of the 7 we need to subtract, how many more do we need to count back? (ability to decompose/partition all numbers applied)

Number Track:



Steps can be recorded on a number track alongside the bead string, prior to transition to number line.

Number Line:



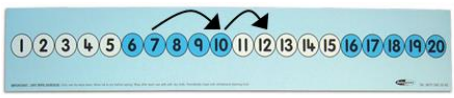
Counting up or 'Shop keepers' method

Bead string:

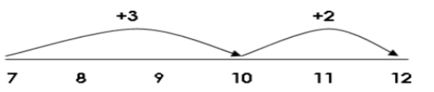
12 – 7 becomes 7 + 3 + 2.

Starting from 7 on the bead string 'how many more to the next multiple of 10?' (children should recognise how their number bonds are being applied), 'how many more to get to 12?'.

Number Track:



Number Line:

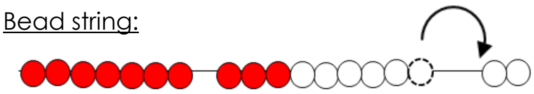


Compensation model (adding 9 and 11)

This model of calculation encourages efficiency and application of known facts (how to add ten)

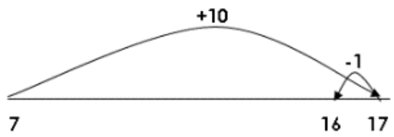
7 + 9

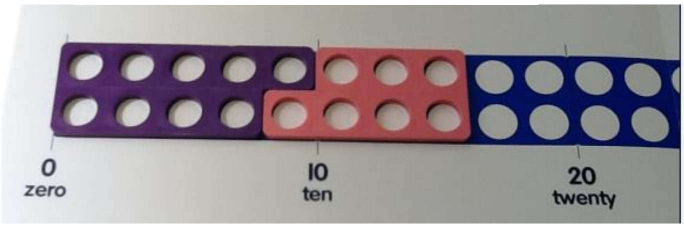
Bead string:



Children find 7, then add on 10 and then adjust by removing 1.

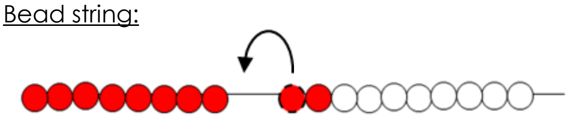
Number line:





18 – 9

Bead string:



Children find 18, then subtract 10 and then adjust by removing 1.

Number line:

Working with larger numbers

TO + TO

Ensure that the children have been transitioned onto Diennes and understand the abstract nature of the single 'tens' sticks and 'hundreds' blocks

Partitioning (Aggregation model)

34 + 23

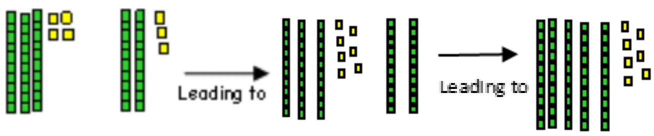
Diennes:

Children create the two sets with Diennes and then combine; ones with ones, tens with tens.

Partitioning (Augmentation model)

Diennes:

Encourage the children to begin counting from the first set of ones and tens, avoiding counting from 1. Beginning with the ones in preparation for formal columnar method.

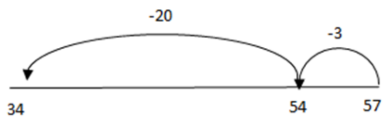


Take away (Separation model)

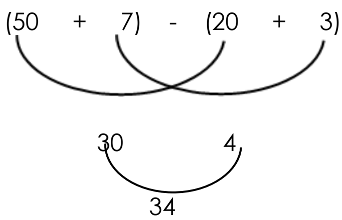
Children remove the lower quantity from the larger set, starting with the ones and then the tens. In preparation for formal decomposition.

57 – 23 = 34

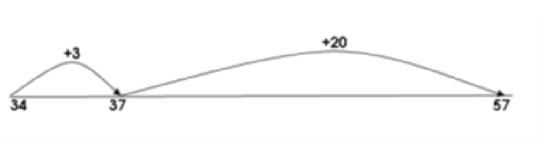
Number Line:



At this stage, children can began to use an informal method to support, record and explain their method.



Number line:



At this stage, children can begin to use an informal method to support, record and explain their method.

30 + 4 + 20 + 3

50 7

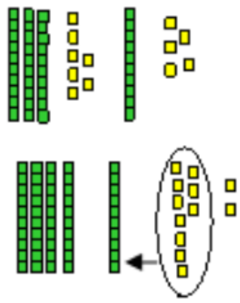
57

Bridging with larger numbers

Once secure in partitioning for addition, children begin to explore exchanging. What happens if the units are greater than 10? Introduce the term 'exchange'. Using the Diennes equipment, children exchange ten ones for a single tens rod, which is equivalent to crossing the tens boundary on the bead string or number line.

Diennes:

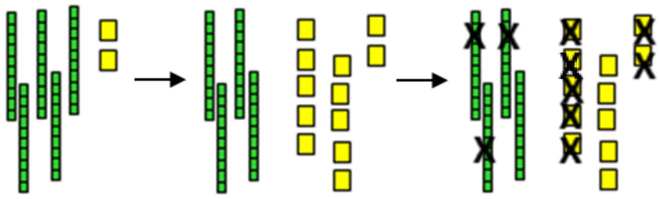
37 + 15



Discuss counting on from the larger number irrespective of the order of the calculation.

Diennes:

52 – 37 = 15



BY THE END OF YEAR 2:

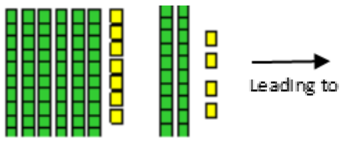
- **Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:**
  - **-a two-digit number and ones**
  - **-a two-digit number and tens**
  - **-two two-digit numbers**
  - **-adding three one-digit numbers**
- **Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot**

Expanded Vertical Method

Children are then introduced to the expanded vertical method to ensure that they make the link between using Diennes equipment, partitioning recording and the expanded vertical method.

Diennes:

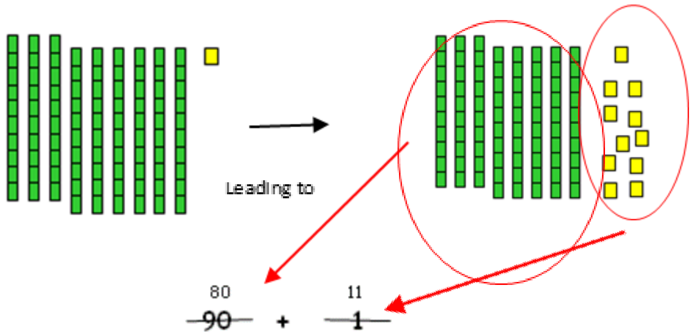
67 + 24 = 91



	67	60 + 7
+	24	20 + 4
	11	7 + 4
	80	60 + 20
	91	

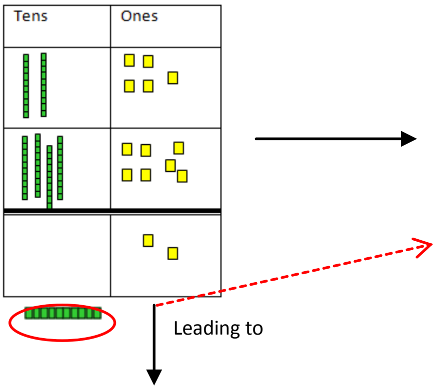
Diennes:

91 – 67 = 24

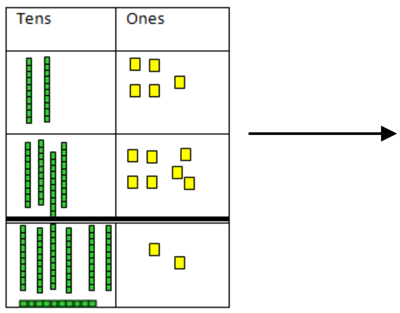


	80		11
	~~90~~	+	~~1~~
-	20	+	4
	60	+	7

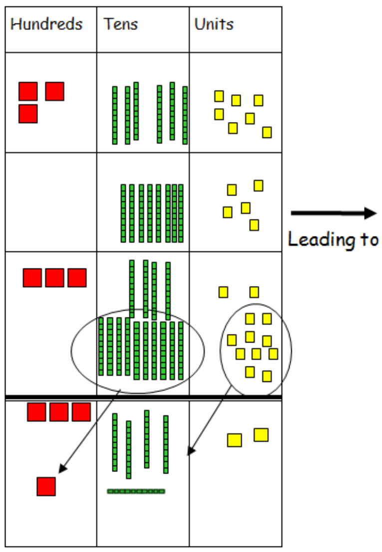
Compact method



25
47
2
1

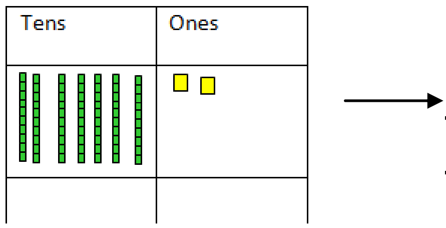


25
47
72
1

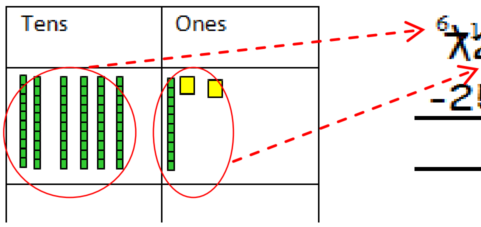


3 6 7
+ 8 5
4 5 2
1 1

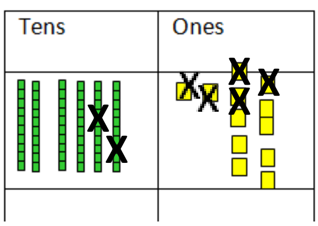
Compact decomposition



72
-25
47



6 1
~~7~~2
-25



6 1
~~7~~2
-25
47

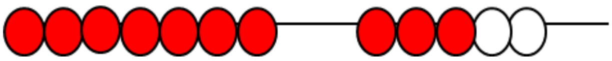
Vertical Acceleration

By returning to earlier manipulative experiences we support children in make links across mathematics, encouraging 'If I know this…then I also know…' thinking.

Decimals

Ensure that children are confident in counting forwards and backwards in decimals – using bead strings to support.

Bead strings:



Each bead represents 0.1, each different block of colour equal to 1.0

Diennes:

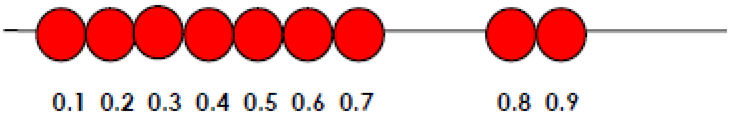
0.1 1.0 10.0

Addition of decimals

Aggregation model of addition

counting both sets– starting at zero.

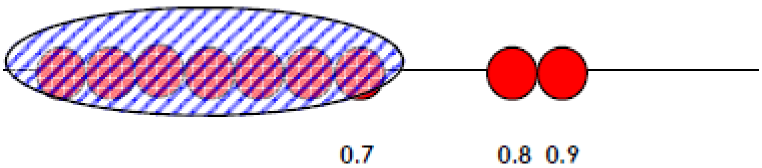
0.7 + 0.2 = 0.9



Augmentation model of addition:

starting from the first set total, count on to the end of the second set.

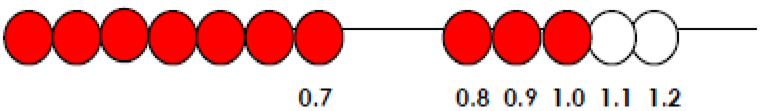
0.7 + 0.2 = 0.9



Bridging through 1.0

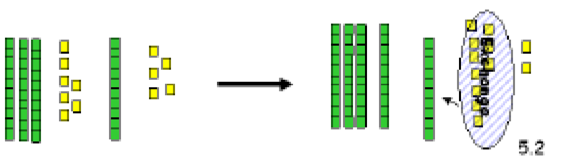
encouraging connections with number bonds.

0.7 + 0.5 = 1.2



Partitioning

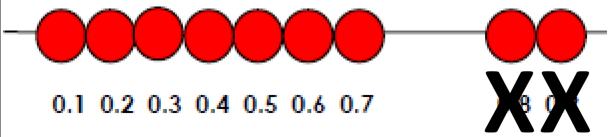
3.7 + 1.5 = 5.2



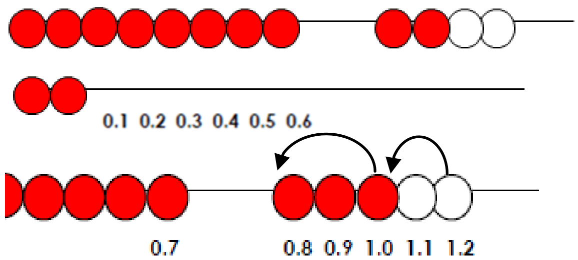
Subtraction of decimals

Take away model

0.9 – 0.2 = 0.7

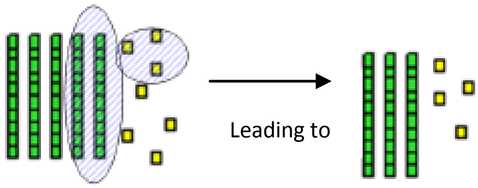


0.8 – 0.2 =



Partitioning

5.7 – 2.3 = 3.4



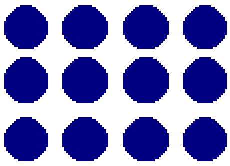
Gradation of difficulty- addition	**Gradation of difficulty- subtraction**
1. No exchange 2. Extra digit in the answer 3. Exchanging units to tens 4. Exchanging tens to hundreds 5. Exchanging units to tens and tens to hundreds 6. More than two numbers in calculation 7. As 6 but with different number of digits 8. Decimals up to 2 decimal places (same number of decimal places) 9. Add two or more decimals with a range of decimal places.	1. No exchange 2. Fewer digits in the answer 3. Exchanging tens for units 4. Exchanging hundreds for tens 5. Exchanging hundreds to tens and tens to units 7. As 6 but with different number of digits 8. Decimals up to 2 decimal places (same number of decimal places) 9. Subtract two or more decimals with a range of decimal places.

Progression in Multiplication and Division

Multiplication and division are connected.

Both express the relationship between a number of equal parts and the whole.

Part	Part	Part	Part
Whole			



The following array, consisting of four columns and three rows, could be used to represent the number sentences: -

3 x 4 = 12, 4 x 3 =12, 3 + 3 + 3 + 3 = 12, 4 + 4 + 4 =12.

And it is also a model for division

12÷4 = 3 12÷3 = 4 12 – 4 – 4 – 4 = 0 12 – 3 – 3 – 3 – 3 = 0

MULTIPLICATION	DIVISION

MULTIPLICATION

Early experiences

Children will have real, practical experiences of handling equal groups of objects and counting in 2s, 10s and 5s. Children work on practical problem solving activities involving equal sets or groups.

DIVISION

Children will understand equal groups and share objects out in play and problem solving. They will count in 2s, 10s and 5s.

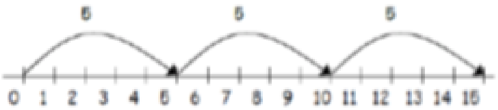
END OF RECEPTION:

- **ELG 11: They solve problems involving doubling, halving and sharing**

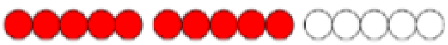
Repeated addition (repeated aggregation)

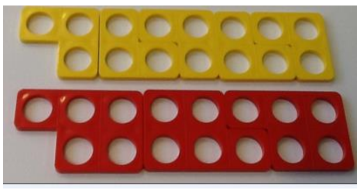
3 times 5 is 5 + 5 + 5 = 15 or 5 lots of 3 or 5 x 3

Children learn that repeated addition can be shown on a number line.



Children learn that repeated addition can be shown on a bead string.

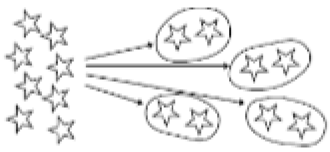




Children also learn to partition totals into equal trains using Cuisenaire Rods

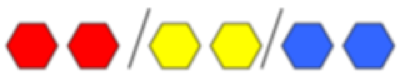
Sharing equally

6 sweets get shared between 2 people. How many sweets do they each get? A bottle of fizzy drink shared equally between 4 glasses.

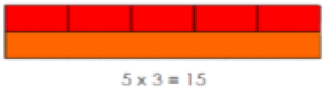


Grouping or repeated subtraction

There are 6 sweets. How many people can have 2 sweets each?



" I have 6 shapes. How many groups of two can I take

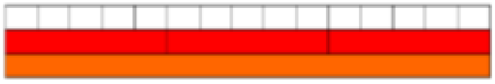


6 ÷2 = 3

Scaling

This is an extension of augmentation in addition, except, with multiplication, we increase the quantity by a scale factor not by a fixed amount. For example, where you have 3 giant marbles and you swap each one for 5 of your friend's small marbles, you will end up with 15 marbles.

This can be written as:
1 + 1 + 1 = 3 →scaled up by 3 →5 + 5 + 5 = 15

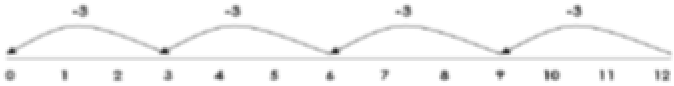


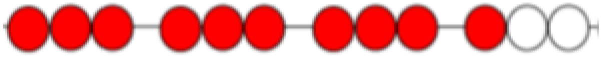
For example, find a ribbon that is 4 times as long as the blue ribbon.

We should also be aware that if we multiply by a number less than 1, this would correspond to a scaling that reduces the size of the quantity. For example, scaling 3 by a factor of 0.5 would reduce it to 1.5, corresponding to 3 X 0.5 = 1.5.

Repeated subtraction using a bead string or number line

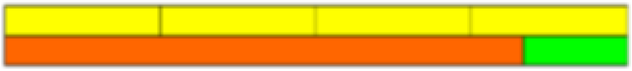
12 ÷ 3 = 4

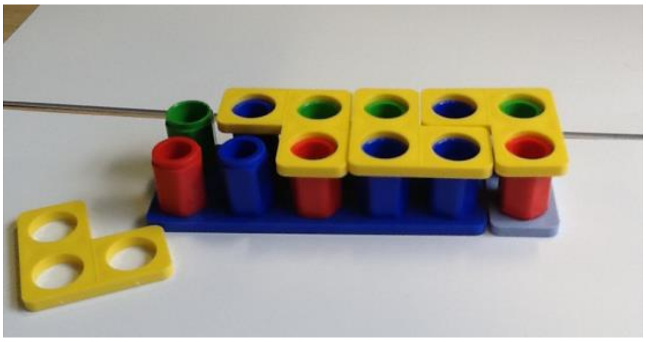




The bead string helps children with interpreting division calculations, recognising that 12 ÷ 3 can be seen as 'how many 3s make 12?'

Cuisenaire Rods also help children to interpret division calculations.

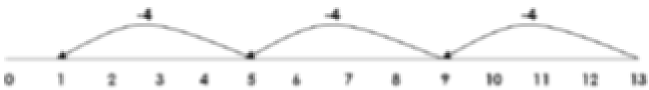




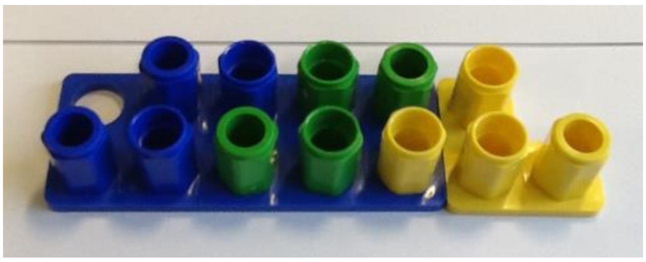
Grouping involving remainders

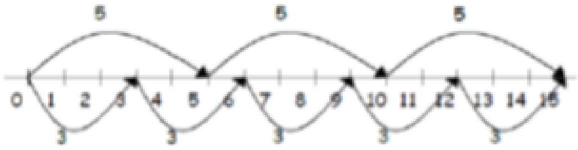
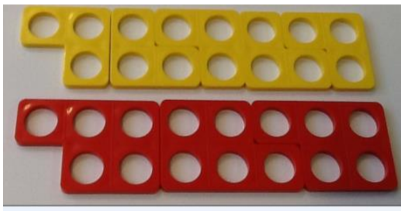
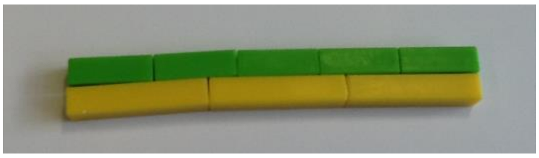
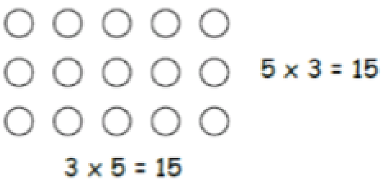
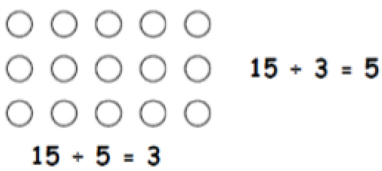
Children move onto calculations involving remainders.

13 ÷ 4 = 3 r1



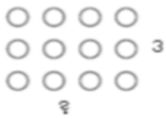
Or using a bead string see above.



	
<u>Commutativity</u> Children learn that 3 x 5 has the same total as 5 x 3. This can also be shown on the number line. 3 x 5 = 15 5 x 3 = 15    	Children learn that division is **not** commutative and link this to subtraction.
<u>Arrays</u> Children learn to model a multiplication calculation using an array. This model supports their understanding of **commutativity** and the development of the grid in a written method. It also supports the finding of factors of a number. 	Children learn to model a division calculation using an array. This model supports their understanding of the development of partitioning and the 'bus stop method' in a written method. This model also connects division to **finding fractions** of discrete quantities. 

Inverse operations

This can be supported using arrays: e.g. 3 X ? = 12



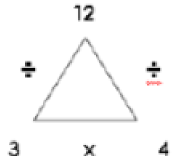
Trios can be used to model the 4 related multiplication and division facts. Children learn to state the 4 related facts.

3 x 4 = 12

4 x 3 = 12

12 ÷ 3 = 4

12 ÷ 4 = 3



Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.

□ x 3 = 12   3 x ∆ = 12   O x □ = 12

12 ÷ 3 = □   12 ÷ O = 3   ∆ ÷ 3 = 4

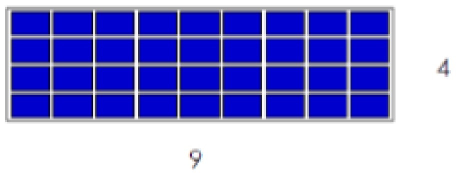
BY THE END OF YEAR 2:

- **Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot**
- **Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.**

Partitioning for multiplication

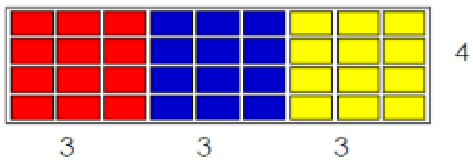
Arrays are also useful to help children visualise how to partition larger numbers into more useful arrays.

9 x 4 = 36



Children should be encouraged to be flexible with how they use number and can be encouraged to break the array into more manageable chunks.

9 x 4 =



Which could also be seen as

9x 4 = (3 x 4) + (3 x 4) + (3 x 4) = 12 + 12 + 12 = 36

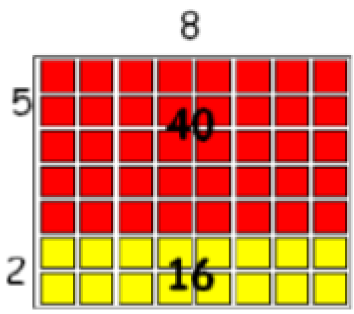
Or 3 x (3x4) = 36

And so 6 x 14 = (2 x 10) + (4 x 10) + (4 x 6) = 20 + 40 + 24 = 84

Partitioning for division

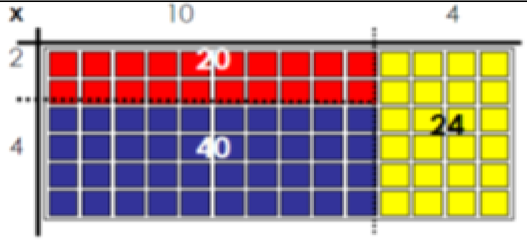
The array is also a flexible model for division of larger numbers

56 ÷ 8 = 7



Children could break this down into more manageable arrays, as well as using their understanding of the inverse relationship between division and multiplication.

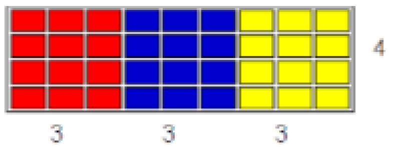
56 ÷ 8 = (40 ÷ 8) + (16 ÷ 8) = 5 + 2 = 7



To be successful in calculation learners must have plenty of experiences of being flexible with partitioning, as this is the basis of distributive and associative law.

Associative law (multiplication only) :-

E.g. 3 x (3x4) = 36

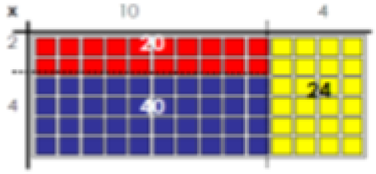


The principle that if there are three numbers to multiply these can be multiplied in any order.

Distributive law (multiplication):-

E.g. 6 x 14 = (2 x 10) + (4 x 10) + (4 x 6) = 20 + 40 + 24 = 84

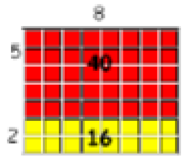
This law allows you to distribute a multiplication across an addition or subtraction.



Distributive law (division):-

E.g. 56 ÷ 8 = (40 ÷ 8) + (16 ÷ 8) = 5 + 2 = 7

This law allows you to distribute a division across an addition or subtraction.



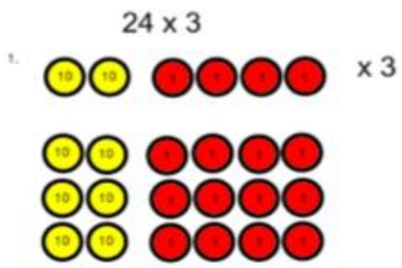
END OF YEAR 4:

- **Multiply two-digit and three-digit numbers by a one-digit number using formal written layout**
- **Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.**

<u>Arrays leading into the grid method</u>

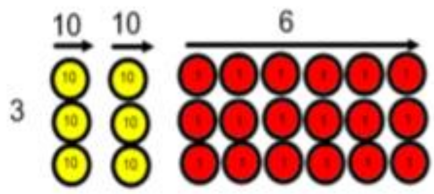
Children continue to use arrays and partitioning where appropriate, to prepare them for the grid method of multiplication.

Arrays can be represented as 'grids' in a shorthand version and by using place value counters we can show multiples of ten, hundred etc.



<u>Arrays leading into chunking and then long and short division</u>

Children continue to use arrays and partitioning where appropriate, to prepare them for the 'chunking' and short method of division. Arrays are represented as 'grids' as a shorthand version.
e.g. 78 ÷ 3 =

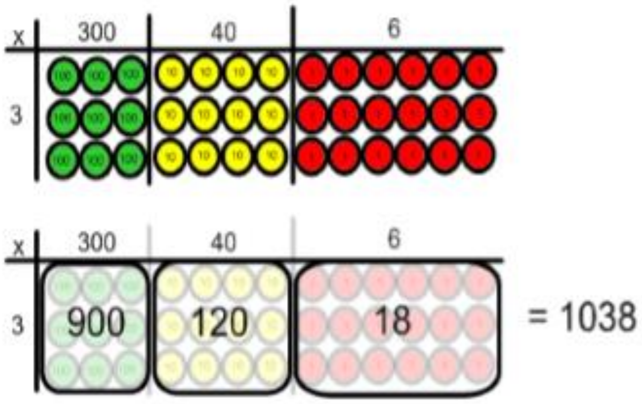


78 ÷ 3 = (30 ÷ 3) + (30 ÷ 3) + (18 ÷ 3) =

10 + 10 + 6 = 26

Grid method

This written strategy is introduced for the multiplication of TU x U to begin with. It may require column addition methods to calculate the total.



The vertical method- 'chunking' leading to long division

See above for example of how this can be modelled as an array using place value counters.

78 ÷ 3 =

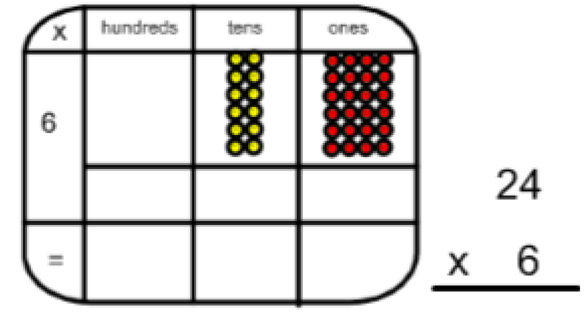
	10	10	6
3	**30**	**30**	**18**

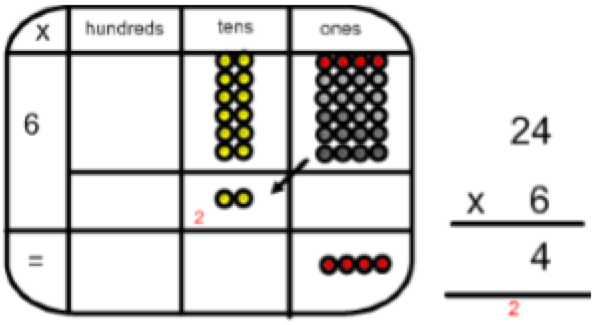
So 78 ÷ 3 = 10 + 10 + 6 = 26

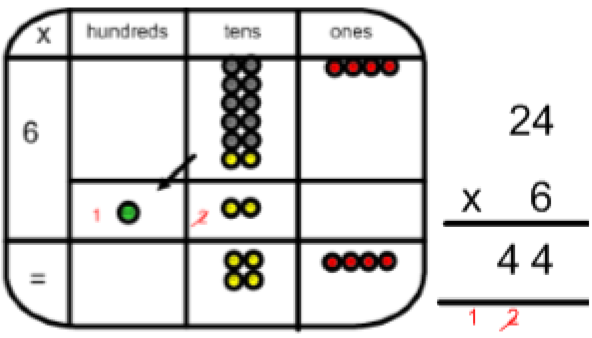
Short multiplication—multiplying by a single digit

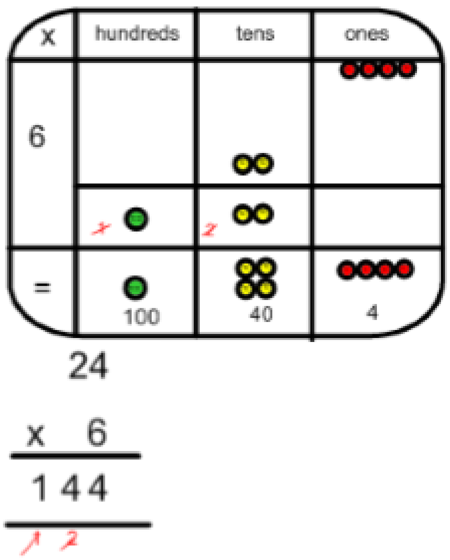
The array using place value counters becomes the basis for understanding short multiplication first without exchange before moving onto exchanging

24 x 6





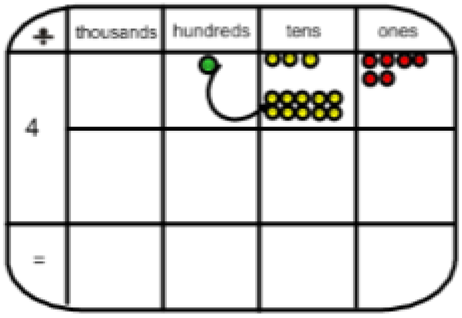


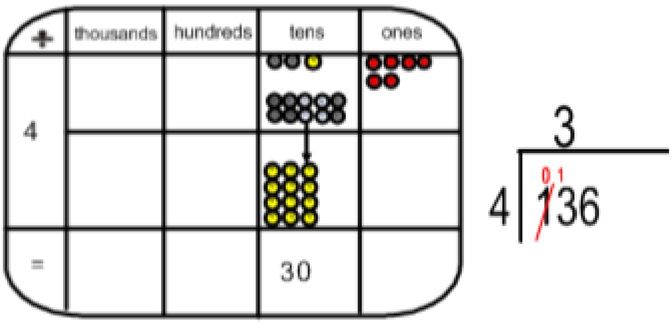


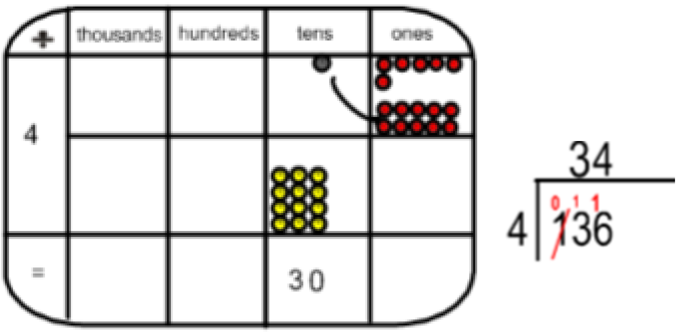
Short division —dividing by a single digit

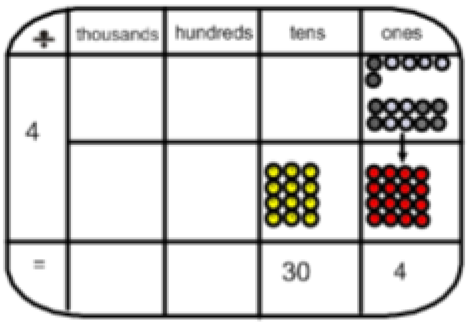
Whereas we can begin to group counters into an array to show short division working

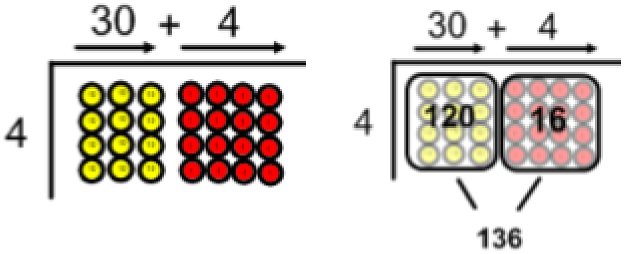
136 ÷ 4





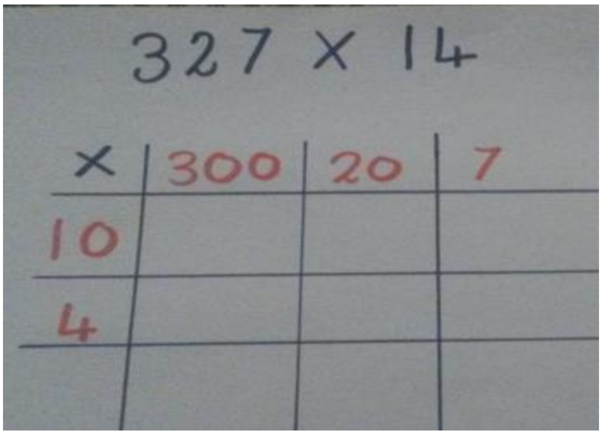


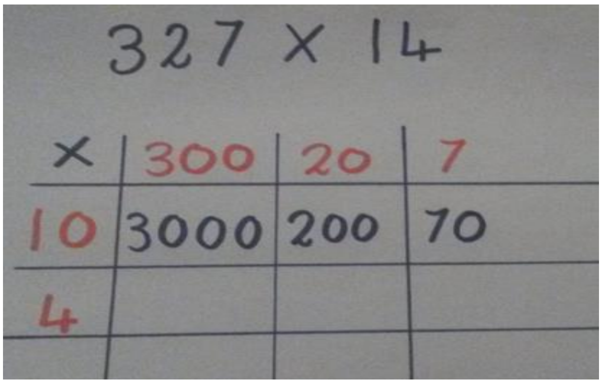


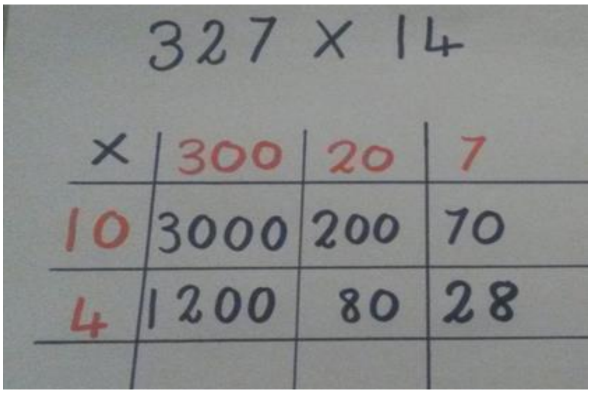


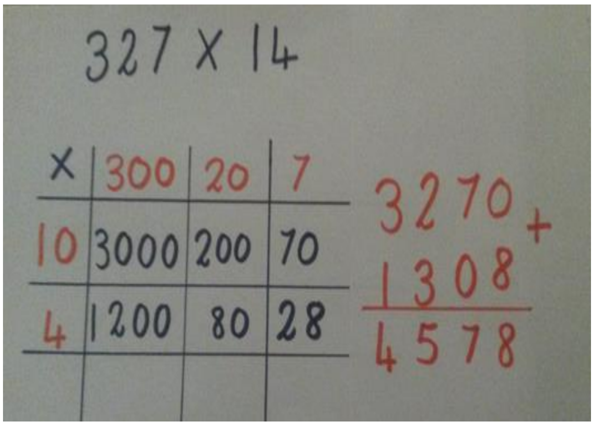
Long multiplication—multiplying by more than one digit

Children will refer back to grid method and compare before being required to record



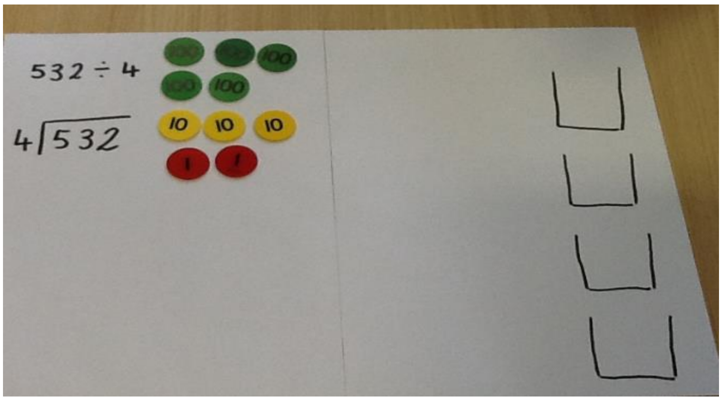


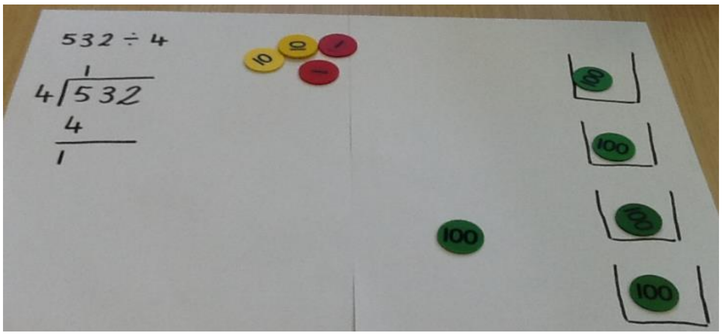


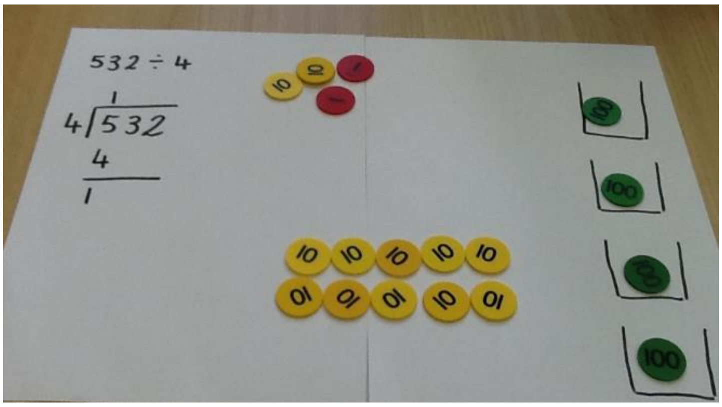


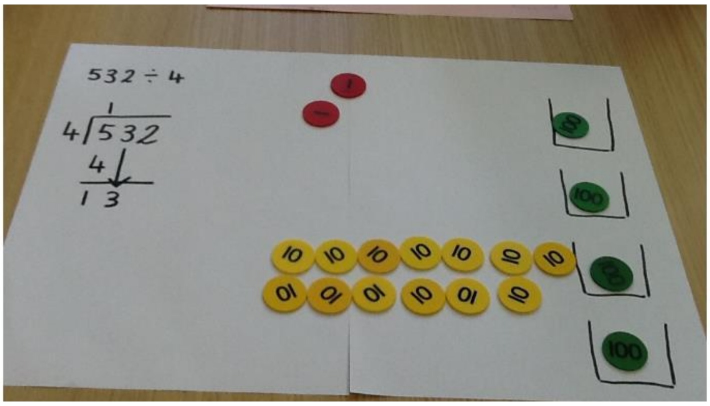
Long division —dividing by more than one digit

Children should be reminded about partitioning numbers into multiples of 10, 100 etc. before recording:

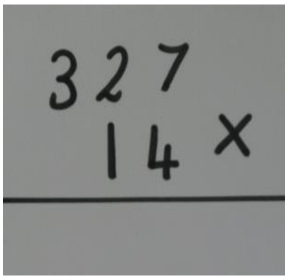


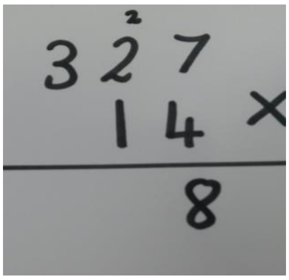


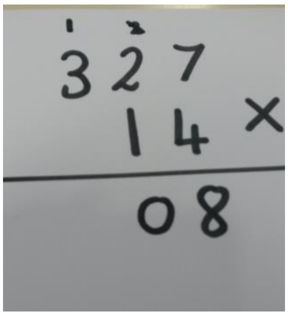


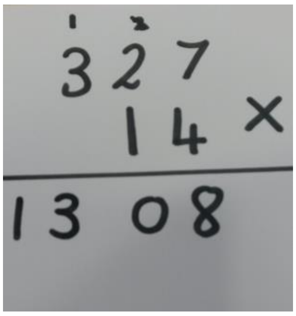


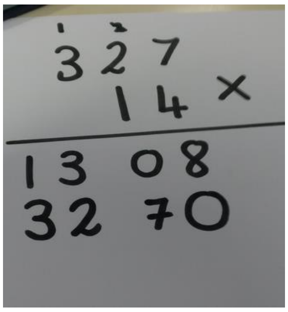
Children will link the grid method to the column method for long multiplication:

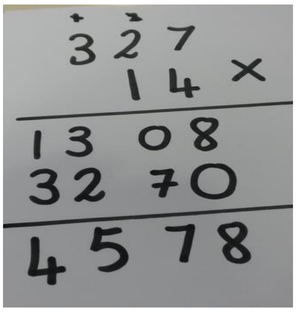


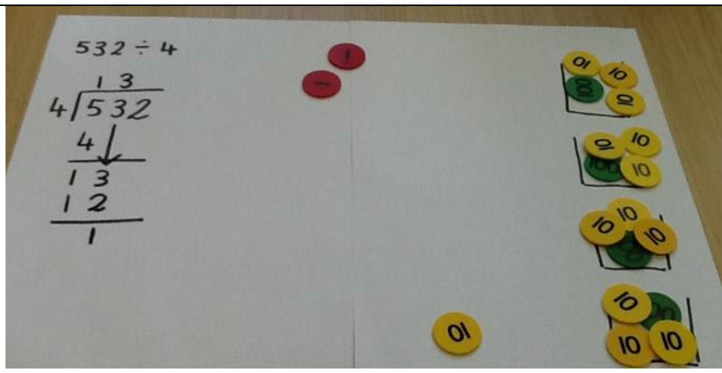


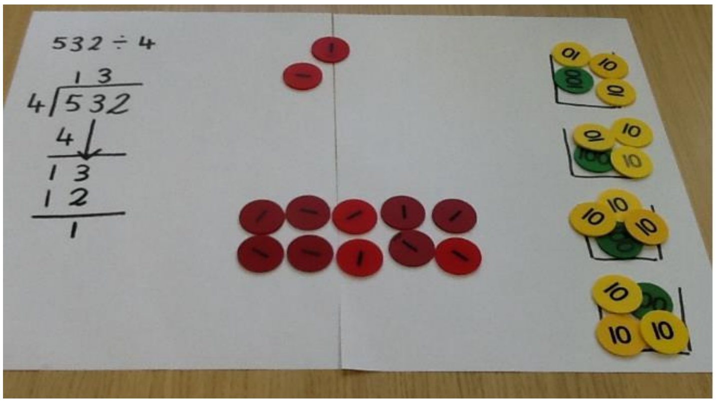


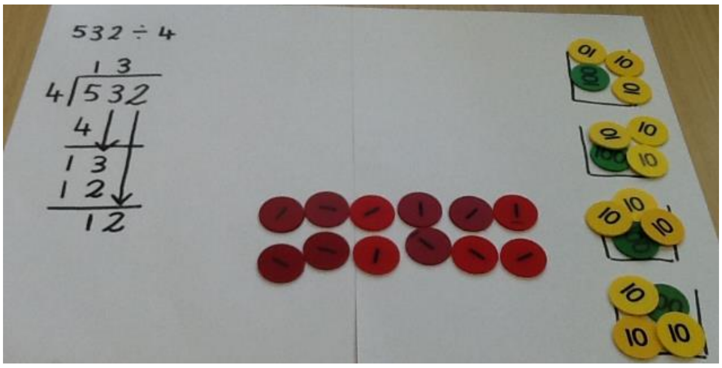












<u>END OF YEAR 6:</u>

- **Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication**
- **Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context**
- **Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context**

Gradation of difficulty (Short multiplication)	Gradation of difficulty (Short division)
1. TO x O no exchange 2. TO x O extra digit in the answer 3. TO x O with exchange of ones into tens 4. HTO x O no exchange 5. HTO x O with exchange of ones into tens 6. HTO x O with exchange of tens into hundreds 7. HTO x O with exchange of ones into tens and tens into hundreds 8. As 4-7 but with greater number digits x O 9. O.t x O no exchange 10. O.t with exchange of tenths to ones 11. As 9 - 10 but with greater number of digits which may include a range of decimal places x O	1. TO ÷ O no exchange no remainder 2. TO ÷ O no exchange with remainder 3. TO ÷ O with exchange no remainder 4. TO ÷ O with exchange, with remainder 5. Zeroes in the quotient e.g. 816 ÷ 4 = **204** 6. As 1-5 HTO ÷ O 7. As 1-5 greater number of digits ÷ O 8. As 1-5 with a decimal dividend e.g. 7.5 ÷ 5 or 0.12 ÷ 3 9. Where the divisor is a two digit number See below for gradation of difficulty with remainders
	Dealing with remainders Remainders should be given as integers, but children need to be able to decide what to do after division, such as rounding up or down accordingly. e.g. · I have 62p. How many 8p sweets can I buy? · Apples are packed in boxes of 8. There are 86 apples. How many boxes are needed? **Gradation of difficulty for expressing remainders** 1. Whole number remainder 2. Remainder expressed as a fraction of the divisor 3. Remainder expressed as a simplified fraction 4. Remainder expressed as a decimal